

QCD sum rules with two-point correlation function

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We construct three different sum rules from the two-point correlation function with pion, $i \int d^4x e^{iq \cdot x} \langle 0 | T J_N(x) \bar{J}_N(0) | \pi(p) \rangle$, beyond the soft-pion limit. The PS and PV coupling schemes in the construction of the phenomenological side are carefully considered in each sum rule. We discuss the dependence of the result on the specific Dirac structure and identify the source of the dependence by making specific models for higher resonances.

Within QCD sum rules, the πNN coupling constant, $g_{\pi N}$, is often calculated [1,2], for example, from the correlation function,

$$i \int d^4x e^{iq \cdot x} \langle 0 | T J_p(x) \bar{J}_n(0) | \pi^+(p) \rangle , \quad (1)$$

where J_p is the proton interpolating field [3] and J_n is the neutron interpolating field. Shiomi and Hatsuda [1] considered $i\gamma_5$ Dirac structure from this correlation function in the soft-pion limit ($p_\mu \rightarrow 0$). Later, Birse and Krippa [2] pointed out that the use of soft-pion limit does not constitute an independent sum rule from the nucleon chiral-odd sum rule, and proposed to look at the Dirac structure, $i\gamma_5 \not{p}$, beyond the soft-pion limit.

Recently [4], we have pointed out that the previous calculations of this sort have dependence on how one models the phenomenological side; either using the pseudoscalar (PS) or the pseudovector (PV) coupling scheme. Beyond the soft-pion limit, we presented a new sum rule for the $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ structure. This sum rule is independent of the coupling schemes and provides $g_{\pi N}$ relatively close to its empirical value.

Then we ask, can we get similar results from the other Dirac structures, $i\gamma_5$ and $i\gamma_5 \not{p}$, constructed beyond the soft-pion limit? If not, what are the reasons for the differences? In this work, we will try to answer these questions by studying all three sum rules and investigating the reliability of each sum rule.

In calculating the OPE of Eq. (1), we only keep the quark-antiquark component of the pion wave function,

$$D_{aa'}^{\alpha\beta} \equiv \langle 0 | u_a^\alpha(x) \bar{d}_{a'}^\beta(0) | \pi^+(p) \rangle , \quad (2)$$

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and use the vacuum saturation hypothesis to factor out higher dimensional operators in terms of the pion wave function and the vacuum expectation value. This quark-antiquark component can be written in terms of the following three matrix elements,

$$\langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 u(x) | \pi^+(p) \rangle, \quad \langle 0 | \bar{d}(0) \gamma_5 \sigma_{\mu\nu} u(x) | \pi^+(p) \rangle, \quad \langle 0 | \bar{d}(0) i \gamma_5 u(x) | \pi^+(p) \rangle, \quad (3)$$

whose few moments are relatively well-known [6].

The first two matrix elements participate in the sum rules with the Dirac structures, $i\gamma_5 \not{p}$ and $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$, while the last matrix element participates only in the $i\gamma_5$ sum rule. Following the standard prescription of QCD sum rule, we obtain for the $i\gamma_5 \not{p}$ structure,

$$g_{\pi N} \lambda_N^2 (1 + AM^2) = \frac{f_\pi}{m} M^2 e^{m^2/M^2} \left[\frac{E_1(x_\pi)}{2\pi^2} M^4 + \frac{E_0(x_\pi)}{2\pi^2} M^2 \delta^2 + \frac{1}{12} \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle + \frac{2\langle \bar{q}q \rangle^2}{9f_\pi^2} \right]. \quad (4)$$

Here $x_\pi = S_\pi/M^2$ with S_π being the continuum threshold and $E_n(x) = 1 - (1 + x + \dots + x^n/n!) e^{-x}$. δ^2 is the twist-4 contribution to the first matrix element in Eq. (3). The unknown single pole, A , contains the contribution of $N \rightarrow N^*$ [5] as well as the PS-PV scheme dependent $N \rightarrow N$ [4]. Thus, physical content of A is coupling-scheme dependent. In obtaining Eq. (4), we have taken out one power of the pion momentum and took the limit $p_\mu \rightarrow 0$ in the rest of the correlator. The corresponding expression in Ref. [2], not only missing the last term in Eq. (4), contains different continuum factors, $E_i(x_\pi)$. The sum rule result strongly depends on these [7].

The sum rule for $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ can be constructed similarly [4],

$$g_{\pi N} \lambda_N^2 (1 + BM^2) = -\frac{\langle \bar{q}q \rangle}{f_\pi} e^{m^2/M^2} \left[\frac{M^4 E_0(x_\pi)}{12\pi^2} + \frac{4}{3} f_\pi^2 M^2 + \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle \frac{1}{216} - \frac{m_0^2 f_\pi^2}{6} \right]. \quad (5)$$

The unknown single pole term is represented by B whose physical content is independent of the coupling schemes. This can be also checked explicitly by constructing B using effective models for higher resonances. m_0^2 is the parameter associated with the dim-5 quark-gluon mixed condensate. We emphasize that this sum rule is independent of the PS and PV coupling scheme employed in the phenomenological side. When this sum rule is combined with the nucleon chiral odd sum rule, it yields $g_{\pi N} \sim 10$ [4].

The sum rule for the $i\gamma_5$ structure beyond the soft-pion limit is complicated due to the PS or PV coupling scheme dependence. By expanding the correlator in terms of the pion momentum, one can construct various sum rules for this structure at each order of the pion momentum. But none of them is independent of the coupling schemes. To achieve the coupling scheme independence, we introduce the kinematical condition,

$$p^2 = 2p \cdot q, \quad (6)$$

which places each nucleon on its mass-shell. In this sum rule, only the last term in Eq. (3) contributes to the OPE side. By keeping up to the order p^2 in the expansion of the correlator in terms of the pion momentum, we obtain,

$$g_{\pi N} \lambda_N^2 (1 + CM^2) = \frac{\langle \bar{q}q \rangle}{f_\pi} e^{m^2/M^2} \left[\frac{0.0785 E_0(x_\pi)}{\pi^2} M^4 - 0.314 \times \frac{1}{24} \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle \right]. \quad (7)$$

Again C denotes the unknown single pole term. Note that this sum rule contains very small numerical factors in the RHS. This is because two independent correlators cancel each other when they are combined via Eq. (6).

The LHSs of the three sum rules, Eqs. (4), (5) and (7), can be written as $c + bM^2$ where c in all sum rules represents $g_{\pi N}\lambda_N^2$ and b denotes the unknown single pole terms whose physical content can be different in each sum rule. We determine b and c by linearly fitting the RHSs within the relevant Borel window and list them in Table 1.

Table 1

The best-fitted values for the parameters c and b obtained within the Borel window $0.8 \leq M^2 \leq 1.2 \text{ GeV}^2$. The continuum threshold $S_\pi = 2.07 \text{ GeV}^2$ is used. To see the sensitivity to the continuum, the results with $S_\pi = 2.57 \text{ GeV}^2$ are also presented in parenthesis. The third column is the difference of the two numbers in the second column.

	$c \text{ (GeV}^6\text{)}$	$ \Delta c \text{ (GeV}^6\text{)}$	$b \text{ (GeV}^4\text{)}$
$i\gamma_5 \not{p}$	-0.00022 (-0.0023)	0.0021	0.011 (0.0145)
$i\gamma_5$	-0.00033 (-0.00016)	0.00017	-0.00183 (-0.0021)
$\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$	0.00308 (0.002906)	0.00017	0.00257 (0.0029)

As shown, the extracted value of $c = g_{\pi N}\lambda_N^2$ is quite different depending on Dirac structures. The $i\gamma_5 \not{p}$ sum rule not only contains the large single pole term (i.e. large b) but is also quite sensitive to the continuum threshold (i.e. large $|\Delta c|$). Therefore, its prediction for c contains large uncertainty due to these effective parameters. The other two sum rules, even though they yield quite different value for c , contain relatively small contribution from the unknown single pole and is less sensitive to the continuum.

The difference in the sensitivity to the continuum threshold as well as in the magnitude of b can be understood by making effective models for the continuum and the unknown single pole [7]. Specifically, using the effective Lagrangians for $N \rightarrow N^*$ and $N^* \rightarrow N^*$, we identify the terms corresponding to the unknown single pole and the step-like continuum. There are two ways to construct the Lagrangians, nonderivative coupling scheme and derivative coupling scheme. It turns out that the $i\gamma_5$ structure within the kinematical condition of Eq. (6) takes the same form as the $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ structure. This explains the similarities between these sum rules. The smallness of b and $|\Delta c|$ can be understood by the cancellation between the positive- and negative- parity higher resonances. This explanation is independent of the coupling schemes.

The large b and $|\Delta c|$ for the $i\gamma_5 \not{p}$ sum rule within the nonderivative coupling scheme can be understood by adding up contributions from the different parity resonances. On the other hand, in the case of the derivative coupling scheme, the additional single pole of $N \rightarrow N$ [4] can explain the large b . In this case, explanation for the strong sensitivity to the continuum threshold is not unique.

Then, why do the $i\gamma_5$ and $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ sum rules lead to different values for c even though they share similar features for the continuum and the unknown single pole? This is an interesting question to pursue in future. At this stage, it is not clear if the difference between the two sum rules is due to the lack of convergence in the OPE or due to the

limitations in the sum rule method itself. In future, it will be interesting to study $i\gamma_5$ sum rule in more detail without imposing Eq. (6). Then the sum rule results clearly depend on the choice of the PS and PV coupling schemes, which can provide further insights into the pion-nucleon coupling.

Nevertheless, our study in this work, though it is specific to the two-point correlation function with pion, raises important issues in applying QCD sum rules in calculating various physical quantities. Sum rules results could have strong dependence on the specific Dirac structure one considers. Similar issue has been raised in Ref. [8] for the case of the nucleon sum rule. Anyway, according to our study, the $i\gamma_5 \not{p}$ structure does not constitute a reliable sum rule as its results are contaminated by the two phenomenological inputs, the unknown single pole and the continuum threshold.

Acknowledgments

This work is supported in part by the Grant-in-Aid for JSPS fellow, and the Grant-in-Aid for scientific research (C) (2) 08640356 of the Ministry of Education, Science, Sports and Culture of Japan. The work of H. Kim is also supported by Research Fellowships of the Japan Society for the Promotion of Science. The work of S. H. Lee is supported by KOSEF through grant no. 971-0204-017-2 and 976-0200-002-2 and by the Korean Ministry of Education through grant no. 98-015-D00061.

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